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Soft-breaking correction to hard supersymmetric relations

—QCD correction to squark decay—

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ABSTRACT

Supersymmetric relations between dimensionless couplings receive finite correction at one-loop when supersymmetry is broken softly. We calculate the $\mathcal{O}(\alpha_s)$ correction to the squark decay width to a quark and an electroweak gaugino, which is found to be nonvanishing. Logarithmic correction appears when the gluino is heavy.

1. Introduction

Supersymmetric field theories comprise a very special subset of general field theories. First of all, a field theory can be supersymmetric only when the number of bosonic and fermionic fields are the same. Supersymmetry thus predicts the existence of superpartners. Moreover, interactions are tightly interrelated if a theory should possess supersymmetry. For example, a selectron couples to a photino and an electron with the coupling strength given by the electromagnetic gauge coupling. If the nature really possesses supersymmetry, discovery of superparticles is thus only the first step to prove it. Verification of supersymmetric relations between various couplings is necessary to establish the theory.

In the minimal supersymmetric standard model (MSSM), supersymmetry is softly broken such that no quadratic divergences appear in mass terms and tadpoles. The breaking consists [1] of the scalar and gaugino mass terms and a certain type of three-scalar couplings (the so-called A term). Superpartners receive a mass of the order of the weak scale from the breaking. Meanwhile, dimension-four interaction terms are not modified, so the selectron-electron-photino coupling retains the value e.

When loop effects are included, however, the soft breaking affects the dimension-four couplings. The general theory of renormalization [2] states that soft symmetry breaking does not generate a new divergence in dimension-four vertices. No new counterterm is called for.[1] Nevertheless, the equality of the couplings prescribed by supersymmetry receives *finite* modification.

In this paper, we examine this effect in a simple example: $\mathcal{O}(\alpha_s)$ correction to the squark-quark-photino coupling. Physically, this vertex can be measured as the decay width of the squark. At the tree level, this width is expressed in terms of the electromagnetic coupling. We will find that there is indeed a finite

1 Soft breaking of supersymmetry is soft in the renormalization theory sense, though the opposite is not true.

$\mathcal{O}(\alpha_s)$ correction to the width. In contrast, the coupling receives no modification if supersymmetry is exact.

Numerous works have been done on calculation of radiative corrections in MSSM, but they are limited to the effects of supersymmetric particles to processes governed by the gauge couplings. To our knowledge, no calculation exists on corrections to hard supersymmetric relations.

Although we refer to photinos most of the time, our result for the correction factor applies to squark decay to a quark and any electroweak gaugino (neutralino or chargino).

2. Lowest-order coupling

The squark-quark-neutralino coupling in MSSM is given by[2]

$$\mathcal{L} = -\sqrt{2}e\Big(n_i^{(L)}\bar{q}\tfrac{1+\gamma_5}{2}\widetilde{\chi}_i^0\tilde{q}_L - n_i^{(R)}\bar{q}\tfrac{1-\gamma_5}{2}\widetilde{\chi}_i^0\tilde{q}_R\Big) + \text{h.c.}\ , \tag{1}$$

where

$$n_i^{(L)} = \big[T_{3L}N_{iz}/\sin\theta_W + (Q - T_{3L})N_{ib}/\cos\theta_W\big]\ , \tag{2a}$$

$$n_i^{(R)} = QN_{ib}^*/\cos\theta_W\ , \tag{2b}$$

and N_{iz}, N_{ib} are the neutralino mixing matrix elements:

$$\widetilde{\chi}_{iL}^0 = N_{ib}\widetilde{B}_L + N_{iz}\widetilde{Z}_L + \text{Higgsino components.} \tag{3}$$

For a photino ($N_{\tilde{\gamma}b} = \cos\theta_W$, $N_{\tilde{\gamma}z} = \sin\theta_W$), one has $n_{\tilde{\gamma}}^{(L)} = n_{\tilde{\gamma}}^{(R)} = Q$. Supersymmetry thus constrains the squark-quark-photino "Yukawa" coupling to be equal (up to a "Clebsch-Gordan" constant) to the electromagnetic gauge coupling e at the tree order, even with soft breaking.

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2 Here we neglect the Yukawa-type interaction of the higgsino component of the neutralino, which is proportional to the quark mass.

The squark decay width to a quark and a neutralino is found to be

$$\Gamma_0(\tilde{q}_L \to q\widetilde{\chi}_i^0) = \tfrac{1}{2}\alpha|n_i^{(L)}|^2 m_{\tilde{q}}(1-r)^2 \ , \tag{4}$$

where $r = m^2_{\widetilde{\chi}^0}/m^2_{\tilde{q}}$. For a photino

$$\Gamma_0(\tilde{q} \to q\widetilde{\gamma}) = \tfrac{1}{2}\alpha Q^2 m_{\tilde{q}}(1-r)^2 \ . \tag{5}$$

The width is determined by the electromagnetic gauge coupling.

3. Comment on the Computational Method

Although supergraph method is powerful in computations with exact supersymmetry, its use seems to be limited in a softly broken theory.[3] We believe that ordinary Feynman graph technique is more convenient. However, there are several complications in practice.

First, supersymmetry should not be violated by regularization. No method is known which fully respects supersymmetry. We use the dimensional reduction method [4] which is compatible with supersymmetry at least at one-loop order.

Second, manifest supersymmetry is lost when we fix the gauge. We work in Wess-Zumino gauge to remove some unphysical fields in the gauge superfield, and further use Feynman gauge to define the gauge field propagator. This has the consequence [5] that the wave function renormalization constant (even the divergent part) for the scalar and the fermion differ from each other. One of its implications is that the usual supersymmetric transformation rule does not hold for the renormalized matter field. A care is thus needed in determining how to

3 The spurion technique to include soft breaking is useful in the calculation of divergent quantities like beta functions [3], or in situations in which the soft breaking can be treated as perturbation.

renormalize the ultraviolet divergence in the squark-quark-photino vertex. Without supersymmetry, this interaction would be an independent coupling on which one could set up any renormalization condition at will. In fact, supersymmetry prescribes the counterterm for the vertex which removes the divergence.

4. $\mathcal{O}(\alpha_s)$ correction under exact supersymmetry

Before discussing the $\mathcal{O}(\alpha_s)$ correction to the squark decay width, we explicitly demonstrate that the equality of $\bar{q}q\gamma$ and $\bar{q}\tilde{q}\widetilde{\gamma}$ coupling is not modified at $\mathcal{O}(\alpha_s)$ if supersymmetry is not broken. We assume the quark and squark have the same mass $m_q \neq 0$, and the gluon, gluino, photon, and photino are massless[4]

The one-loop graphs for the $\bar{q}\tilde{q}\widetilde{\gamma}$ coupling is shown in Fig. 1. We evaluate these diagrams at the "on-shell" limit: the quark (squark) are on their mass shell, and the four-momentum squared q^2 of the photino is taken to be $q^2 \to 0$. For the $\bar{q}q\gamma$ vertex n the corresponding limit, there is no $\mathcal{O}(\alpha_s)$ correction in the on-shell renormalization scheme. We use dimensional reduction with $D = 4 - 2\epsilon$ for ultraviolet cutoff and regularize infrared divergences by an infinitesimal gluon mass λ. The gluon exchange diagram gives

$$\frac{C_F \alpha_s}{4\pi} \left[\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2} - 2 \log \frac{m^2}{\lambda^2} + 4 \right] \times (\text{lowest}) \; , \tag{6}$$

and the gluino exchange contributes

$$\frac{C_F \alpha_s}{4\pi} \cdot 2 \times (\text{lowest}) \; . \tag{7}$$

Here $C_F = 4/3$ is a color factor and μ is the arbitrary renormalization scale.[5] The sum of these contribution is both ultraviolet and infrared divergent. The

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4 Here the quark and squark can be thought to have gauge-invariant masses, because only strong and electromagnetic couplings enter at the order we work. The corrections discussed below are thus identical for $\tilde{q}_L$ and $\tilde{q}_R$.

5 In the usual convention, our $1/\epsilon$ should be read as $1/\epsilon - \gamma_E + \ln 4\pi$.

necessary counterterm to render it finite may be found as follows. At $\mathcal{O}(\alpha_s)$, neither the QED coupling e nor the photino field receives corrections. The counterterm is then determined by the wave function renormalization for the quark and squark fields to be

$$\left(Z_q^{1/2} Z_{\tilde{q}}^{1/2} - 1\right) \times (\text{lowest order vertex})\ . \tag{8}$$

Here Z_q ($Z_{\tilde{q}}$) is the wave function renormalization constant for the quark (squark). We evaluate the renormalization constants in the on-shell renormalization scheme from the quark and squark two-point functions. The diagrams needed are shown in Fig. 2. We find

$$Z_q - 1 = \frac{C_F \alpha_s}{4\pi} \left[-\frac{2}{\epsilon} + 2\log\frac{m^2}{\mu^2} + 2\log\frac{m^2}{\lambda^2} - 8 \right] , \tag{9a}$$

$$Z_{\tilde{q}} - 1 = \frac{C_F \alpha_s}{4\pi} \left[2\log\frac{m^2}{\lambda^2} - 4 \right] , \tag{9b}$$

The two counterterms are not equal because our calculation is in the Wess-Zumino-Feynman gauge which is not manifestly symmetric.[6] The counterterm contribution (8) exactly cancels the one-loop graphs (6) and (7). Therefore, the supersymmetric relation between the $\bar{q}q\gamma$ and $\bar{q}\tilde{q}\widetilde{\gamma}$ couplings receives no correction at $\mathcal{O}(\alpha_s)$ when supersymmetry is exact.[7]

6 The mass counterterms are found to satisfy the manifestly supersymmetric relation $\delta m_{\tilde{q}}^2 = 2m_q \delta m_q$.

7 This result can be verified using supersymmetric Ward identity [6].

5. $\mathcal{O}(\alpha_s)$ correction to the squark decay width

Now we turn on the soft supersymmetry breaking which shifts upward the mass of the squark, gluino, and photino. We assume $m_{\tilde{\gamma}} < m_{\tilde{q}}$ so that the on-shell process $\tilde{q} \to q\widetilde{\gamma}$ is kinematically allowed. Although we consider the decay $\tilde{q}_L \to q\widetilde{\gamma}$ for definiteness, the result is the same for $\tilde{q}_R$ (after the exchange $L \leftrightarrow R$).

The $\mathcal{O}(\alpha_s)$ contribution to the $\tilde{q}_L q\widetilde{\gamma}$ vertex comes from the diagrams in Fig. 1 plus the counterterm. Each contribution is proportional to the lowest order vertex (there is only one Lorentz structure for the vertex because of chirality conservation).

Real gluon emission $\tilde{q}_L \to qg\widetilde{\gamma}$ appears at the same order and must be added to the total rate to cancel infrared divergence. There are two diagrams for this process (see Fig. 3).

The total decay rate up to $\mathcal{O}(\alpha_s)$ can be written as

$$\Gamma = \Gamma_0\left(1 + \frac{C_F\alpha_s}{\pi}F\right), \tag{10}$$

with

$$F = F_g + F_{\tilde{g}} + F_{\rm ren} + F_{\rm real}\,. \tag{11}$$

Here F_g, *etc.*, are the contributions of Fig. 1(a), (b), the counterterm, and the real gluon emission respectively.

For clarity, we neglect the mass of the photino for a while. The gluon-exchange (Fig. 1(a)) gives

$$F_g = \frac{1}{2\epsilon} - \frac{1}{2}\log\frac{m_{\tilde{q}}^2}{\mu^2} - \frac{1}{4}\log^2\delta - \log\delta - \frac{\pi^2}{4}\,. \tag{12}$$

Here $\delta = \lambda^2/m_{\tilde{q}}^2$,

There are two diagrams with a gluino in the loop, one with a $\tilde{q}_L$, another with a $\tilde{q}_R$. In the massless quark limit, $\tilde{q}_R$ does not contribute because of chirality conservation. It turns out that the diagram with $\tilde{q}_L$ also vanishes. This is due to crash between the Lorentz and chirality structure of the graph. Hence $F_{\tilde{g}} = 0$.

The real gluon emission, integrated over the whole phase space, gives

$$F_{\rm real} = \frac{1}{4}\log^2\delta + \frac{5}{4}\log\delta + \frac{13}{4} - \frac{\pi^2}{12}\,. \tag{13}$$

The counterterm is defined in the same way as in the supersymmetric case. We calculate the quark and squark two-point functions and find

$$\begin{aligned} F_{\rm ren} &= \left(\frac{C_F\alpha_s}{\pi}\right)^{-1}\big[(Z_q-1)+(Z_{\tilde{q}}-1)\big] \\ &= -\frac{1}{2\epsilon} + \frac{1}{2}\log\frac{m_{\tilde{q}}^2}{\mu^2} - \frac{1}{4}\log\delta + \frac{3}{4}\log R - \frac{1}{2}R - 1 \\ &\quad -\frac{1}{2}(R^2-1)\log\frac{|R-1|}{R} + \frac{1}{4}\left[\frac{2R-1}{(R-1)^2}\log R - \frac{1}{R-1}\right]\,, \end{aligned} \tag{14}$$

where $R = m_{\tilde{g}}^2/m_{\tilde{q}}^2$.

This contribution cancels both ultraviolet and infrared divergences, but a finite correction remains. The total correction factor is[8]

$$F = \frac{3R^2-4R+2}{4(R-1)^2}\log R - \frac{1}{2}(R^2-1)\log\frac{|R-1|}{R} - \frac{2R^2-11R+10}{4(R-1)} - \frac{\pi^2}{3}\,. \tag{15}$$

Interestingly, (15) depends on the gluino mass even though the loop graph with a gluino vanishes. The dependence comes from wave function renormalization. The mass dependence of the correction factor is shown in Fig. 4 (solid

8 As an alternative renormalizational procedure, we may use minimal subtraction ($\overline{\rm MS}$) to renormalize the vertex as well as the propagators (the counterterms just consist of poles in ϵ). The physical S matrix is then obtained by LSZ reduction with the inclusion of the finite wave function renormalization factor. The total result is identical with (15).

curve). The lowest order rate changes by about 5% (for $\alpha_s \sim 0.1$) if the gluino is not too heavy. In particular, $F(R=0) = \frac{5}{2} - \frac{\pi^2}{3} \simeq -0.790$ and $F(R=1) = \frac{17}{8} - \frac{\pi^2}{3} \simeq -1.165$. At the heavy gluino limit, we find

$$F \simeq \frac{3}{4} \log \frac{m_{\tilde{g}}^2}{m_{\tilde{q}}^2} + \frac{5}{2} - \frac{\pi^2}{3} \ . \tag{16}$$

This logarithmic behavior can be understood in the following way. Without supersymmetry, the photino coupling e in (1) is a Yukawa-type coupling independent of the electromagnetic gauge coupling. If we denote the former coupling by f, exact supersymmetry demands $f = e$. When $m_{\tilde{g}} \gg m_{\tilde{q}}$, supersymmetry is broken at the gluino mass scale, below which f and e need not be equal. In fact, the $\mathcal{O}(\alpha_s)$ renormalization group equation for f below $m_{\tilde{g}}$ is found to be

$$\frac{df}{d\log\mu^2} = -\frac{3C_F}{8\pi}\,\alpha_s f \ , \tag{17}$$

whereas the gauge coupling e does not run at $\mathcal{O}(\alpha_s)$. It can be seen that the logarithmic correction found in the full calculation is nothing but the effect of the running of f from the gluino mass to the squark mass, the typical energy for the decay process.

Finally, we calculate the correction for massive photino $0 < r < 1$, which may be practially important. The diagram with a gluino is now nonzero (no divergence appears because the amplitude is proportional to $m_{\tilde{\gamma}}$ as well as $m_{\tilde{g}}$). We find

$$F_g = \frac{1}{2\epsilon} - \frac{1}{2}\log\frac{m_{\tilde{q}}^2}{\mu^2} - \frac{1}{4}\log^2\frac{\delta}{(1-r)^2} - \log\delta - \mathrm{Li}_2\,(r) + \log(1-r) - \frac{\pi^2}{4} \ , \tag{18}$$

$$F_{\tilde{g}} = -\sqrt{Rr}\left\{ \frac{R+r-2}{(1-r)^2} H(R,r) + \frac{1}{r}\log(1-r) + \frac{1}{1-r}\big[R\log R - (R-1)\log|R-1|\big] \right\} , \tag{19}$$

$$F_{\text{real}} = \frac{1}{4}\log^2\frac{\delta}{(1-r)^2} - \mathrm{Li}_2\,(r) - \log r \log(1-r) - \frac{\pi^2}{12}$$
$$+ \frac{5}{4}\log\delta - \frac{5}{2}\log(1-r) - \frac{r(4-3r)}{4(1-r)^2}\log r + \frac{13-14r}{4(1-r)}\,. \tag{20}$$

The function H is given by

$$\begin{aligned} H(R,r) =& \mathrm{Li}_2\left(\frac{R-1}{Rr-1}\right) - \mathrm{Li}_2\left(\frac{R+r-2}{Rr-1}\right) - \mathrm{Li}_2\left(\frac{r(R-1)}{Rr-1}\right) \\ &+ \mathrm{Li}_2\left(\frac{r(R+r-2)}{Rr-1}\right) \quad (\text{for } Rr<1) \\ =& -\mathrm{Li}_2\left(\frac{Rr-1}{R-1}\right) + \mathrm{Li}_2\left(\frac{Rr-1}{R+r-2}\right) + \mathrm{Li}_2\left(\frac{Rr-1}{r(R-1)}\right) \\ &- \mathrm{Li}_2\left(\frac{Rr-1}{r(R+r-2)}\right) - \log r \log\frac{R+r-2}{R-1} \quad (\text{for } Rr>1) \end{aligned} \tag{21}$$

The counterterm contribution F_{ren} is given by (14).

The dependence of the result on $m_{\tilde{\gamma}}$ is shown in Figs. 4 and 5. The logarithmic singularity at $r \to 1$ is killed by the phase space factor in the decay rate.

6. Conclusion

We have calculated the $\mathcal{O}(\alpha_s)$ correction to the squark decay width to a quark and an electroweak gaugino. We have found that the correction is nonzero, which can be interpreted as a manifestation of the soft supersymmetry breaking. In particular, logarithmic correction appears if the gluino is heavier than the squark. This has an interesting implication that the supersymmetry breaking scale may be inferred from observables at much lower energies, because supersymmetry provides a boundary condition to connect couplings which are otherwise unrelated. Unfortunately, this particular example is not very realistic in supergravity-motivated models, in which the gluino cannot be much heavier than the squarks, with the possible exceptions of the scalar top and bottom.

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Figure Captions

Fig. 1. One-loop diagrams for the $\bar{q}\tilde{q}\widetilde{\gamma}$ vertex. The arrow shows the flow of quark number.

Fig. 2. One-loop diagrams for (a) quark and (b) squark self energies.

Fig. 3. Feynman diagrams for $\tilde{q} \to q\widetilde{\gamma}g$.

Fig. 4. $m_{\tilde{g}}$ dependence of the correction factor F for massless photino (solid), $m_{\tilde{\gamma}}/m_{\tilde{q}} = 0.2$ (dash), 0.5 (dashdot), and 0.9 (dot).

Fig. 5. $m_{\tilde{\gamma}}$ dependence of the correction factor F for $m_{\tilde{g}}/m_{\tilde{q}} = 0.1$ (dash), 1 (solid), 3 (dot).

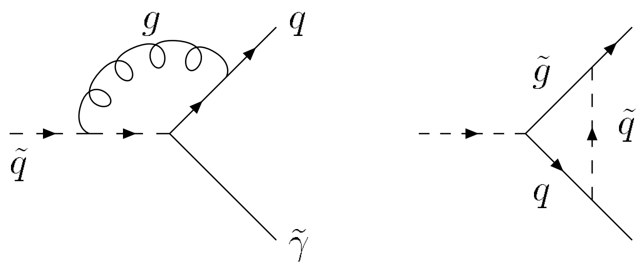
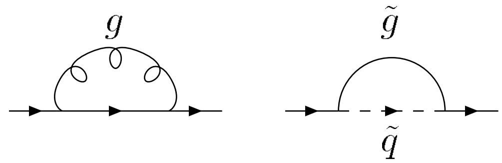

g
q
$\tilde{q}$
$\tilde{\gamma}$
$\tilde{g}$
$\tilde{q}$
q

Fig. 1


g
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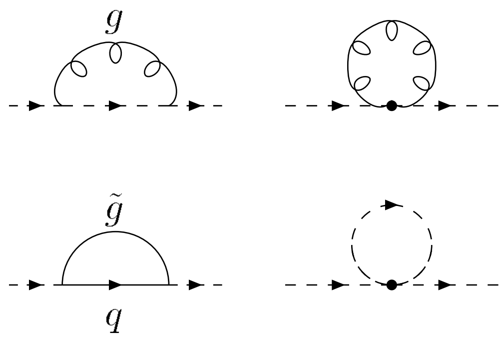
$\tilde{q}$

(a)


g
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(b)

Fig. 2

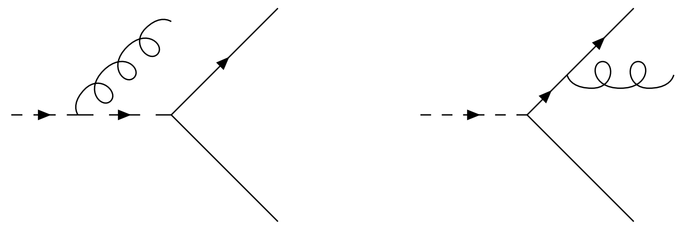

Fig. 3

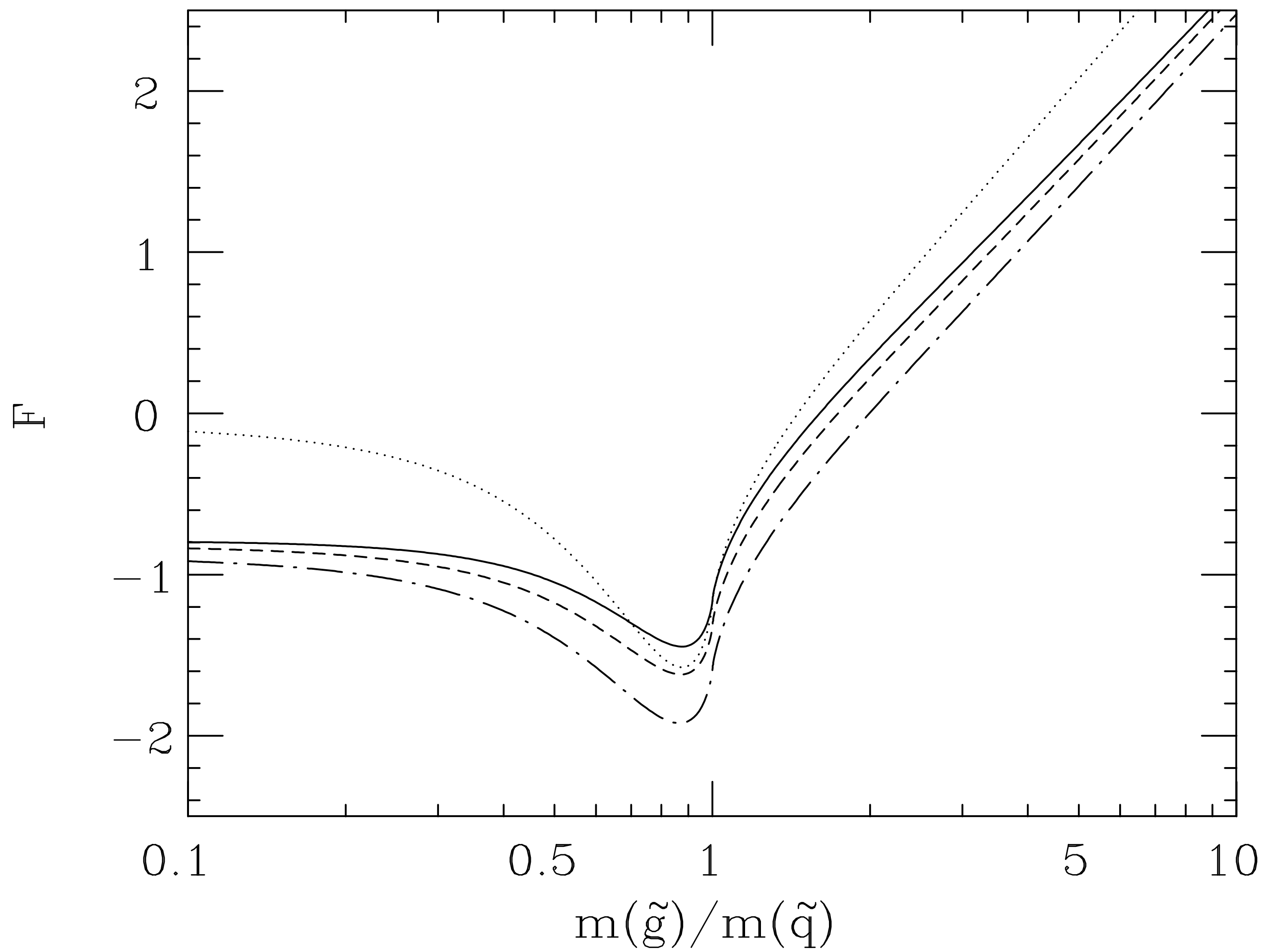

Fig. 4

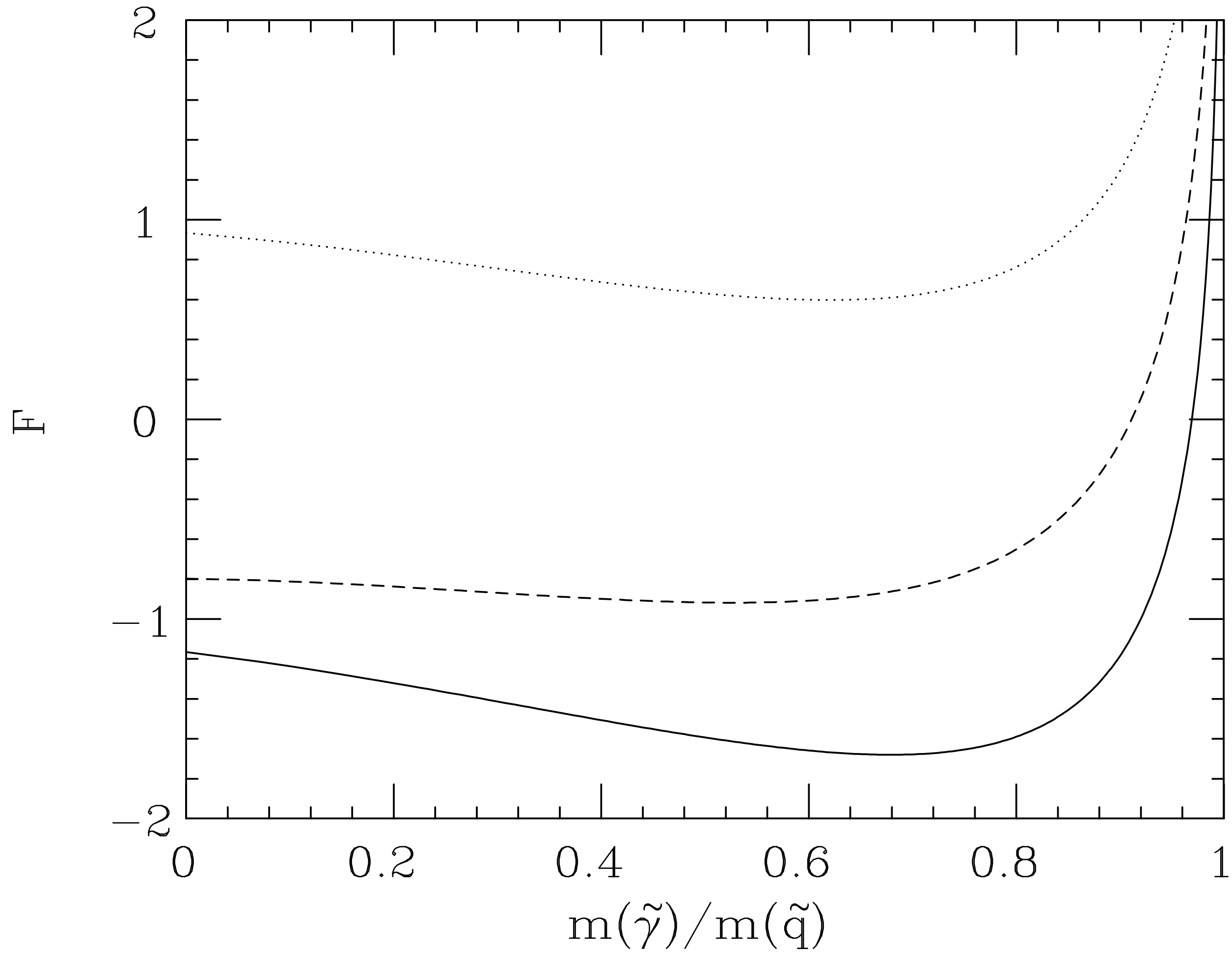

Fig. 5